

# MSCS Mess

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## Problem of the Week

This week's problem comes from Professor Blanchard, and is based on a research project of his. There are two versions of the problem, the first of which is much more friendly than the second:

### Friendly:

The set  $A = \{0, 2, 3, 4, 12\}$  has property P: For every subset  $S$  with 2 or more elements, the smallest difference between elements of  $S$  divides every other difference between elements of  $S$ . For example, in the subset  $B = \{0, 3, 12\}$ , the smallest difference 3 divides the other two differences 9 and 12. Find the smallest positive integer  $x$  not already in  $A$  such that  $A \cup \{x\}$  also has property P.

### Very Difficult:

$A = \{1, 4001, 4501, 5001, 6001\}$   
Using notation as defined above, find the smallest positive integer  $x$  not already in  $A$  such that  $A \cup \{x\}$  also has property P.

Solutions can be e-mailed to Professor Blanchard ([blanchap@stolaf.edu](mailto:blanchap@stolaf.edu)).

## Millenium Prize Problem: The Poincaré conjecture

This theorem was conjectured by Henri Poincaré, a French mathematician and theoretical physicist, in the beginning of the 20<sup>th</sup> century. It is a theorem in topology that states:

Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.

Now, what does this mean? Let's break down this theorem into more easily accessible terms. Imagine a rubberband. Now take that rubberband and stretch it around a sphere. Since we can essentially shrink the rubberband down to a point without tearing the sphere or the rubberband, and without leaving the surface, we say that a sphere is **simply connected**. Now imagine a donut in 3-space. A donut is not simply connected because it is impossible to slowly shrink the rubberband to a single point without either leaving the donut or breaking the rubber band or donut. Thus, a donut *is not* simply connected. The 3-sphere is like a regular sphere (2-sphere), but extended into a higher dimension. The 2-sphere can be thought of as a surface that forms the boundary of a ball in 3-dimensions. Likewise, the 3-sphere can be thought of as a surface that forms the boundary of a ball in 4-dimensions. Lastly,

a homeomorphism can be thought of as the isomorphisms in topological spaces. So for example, if two groups are isomorphic, then they have essentially the same structure and can be looked at as identical. With topological spaces, a homeomorphism is a continuous function between topological spaces that has a continuous inverse function. Thus, you can essentially deform one object into another. If two objects have a homeomorphism between them, then they are looked at as essentially the same in topology.

This problem is the only problem of the set to have been solved. The solution was presented by Grigori Perelman (a Russian mathematician) in 2002 and 2003 in three papers. It took mathematicians 3 years to confirm this solution as correct. Once everyone agreed he had come up with a correct solution, he was awarded the \$1,000,000 prize along with the Fields Medal (essentially the Nobel Prize of Mathematics awarded every 4 years). However, he declined both the prize money and Fields Medal stating that he did not think his contribution to the solution was any greater than another mathematicians (Richard Hamilton).

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